LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2013**

ST 3503/3501/3500 - STATISTICAL MATHEMATICS - II

Date: 29/04/2013 Dept. No. Max. : 100 Marks

Time: 9:00 - 12:00

PART – A

Answer ALL the questions: (10 x 2 = 20)

1. State any two properties of the Riemann integral.
2. Write down the properties of probability density function (p.d.f.)
3. State Comparison test for an improper integral of continuous functions.
4. Define Beta function and prove the symmetry of Beta function.
5. A continuous random variable X has the p.d.f. $f(x) = 3x^2$, $0 \le x \le 1$. Find the value of 'a' such that $P(X \le a) = P(X > a)$. Find the value of 'b' such that $P(X > b) = 0.05$
6. State the conditions under which the Laplace transform of f(t) exists.
7. Find the order and degree of the differential equation $y = xy'' + r\sqrt{1+(y')^2}$.
8. Shade the region of integration in $I = \int_0^1 \int_0^x dy\, dx$.
9. State Cayley- Hamilton Theorem and give two of its applications.
10. What are the Eigen values of $\begin{bmatrix} 1 & 7 & 5 \\ 0 & 2 & 9 \\ 0 & 0 & 5 \end{bmatrix}$.

PART – B

Answer any FIVE questions: (5 x 8 = 40)

11. Let f be a continuous real valued function on the closed bounded interval [a,b]. If the maximum value for f is attained at $c \in (a, b)$ and if $f'(c)$ exists, show that $f'(c) = 0$.

12. If f is a continuous function in the closed bounded interval [a,b] and if $\varphi'(x) = f(x)$, for $a \le x \le b$, then show that $\int_a^b f(x)\,dx = \varphi(b) - \varphi(a)$.

13. Prove that the improper integrals

    (i) $\int_1^\infty \frac{1}{x}\,dx$ diverges (ii) $\int_1^\infty \frac{1}{\sqrt{1-x^2}}\,dx$ absolutely converges.

14. Using Beta and Gamma functions, evaluate (i) $\int_1^2 x(8-x^3)^{1/3}\,dx$ (ii) $\int_0^\infty \frac{1}{1+x^4}\,dx$.

15. **Find the m.g.f, mean and variance of the distribution whose pdf is**

$f(x) = \begin{cases} k e^{-kx}, & x > 0 \\ 0 \; , & otherwise \end{cases}$. **Hence, find mean, variance,** μ'_3, μ'_4.

16. **Solve** $(D^2 + 5D + 4)\, y = x^2 + 7x + 9$.

17. **Evaluate** $\iint \frac{x^2 y^2}{x^2 + y^2}\, dx\, dy$ **over the region between the circles** $x^2 + y^2 = a^2$ **and** $x^2 + y^2 = b^2$

(b > a) by transforming into polar coordinates.

18. **Show that for a square matrix,**

a) '0' is a characteristic root of a matrix if the matrix is singular.

b) The characteristic root of a real symmetric matrix are real.

c) Every Eigen vector corresponds to a unique Eigen value.

PART – C

Answer any TWO questions: (2 x 20 = 40)

19. **a) If** $f \in R[a, b]$**, and** $a < c < b$**, then show that** $f \in R[a, c]$, $f \in R[c ,b]$ **and that**

$\int_a^b f = \int_a^c f + \int_c^b f$.

b) State and prove the first fundamental theorem of integral calculus.

20. **a) The joint pdf of the random variables (X, Y) is** $f(x, y) = k(x + y)$, $0 \le x,\ y \le 1$. **Find 'k' and Cov(X, Y).**

b) Show that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$.

21. **a) If L{f (t)} = F(s), then prove that L** $\left\{\frac{1}{t} f(t)\right\} = \int_s^\infty F(s)\, ds$ **provided the integral exists.**

Also Find Laplace transform of $\frac{Sin\ at}{t}$.

b) Solve $(D^2 - 4D + 3)\, y = Sin\ 3x\ \ Cos\ 2x$.

22. **a) Verify whether the following system of equations is consistent. If consistent, find the solutions.**

$x - 4y - 3z = -16$, $4x - y + 6z = 16$

$2x + 7y + 12z = 48$, $5x - 5y + 3z = 0$.

b) Verify Cayley Hamilton theorem and hence find

A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

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